

Physics 4A

Chapters 12: Rotation of a Rigid Body

“Nothing can bring you peace but yourself.” – Ralph Waldo Emerson

“The foolish man seeks happiness in the distance, the wise man grows it under his feet.”
James Oppenheim

“Happiness is not a station you arrive at, but a manner of traveling.” – Margaret B. Runbeck

Reading: pages 92 – 102; 294 – 324

Outline:

- ⇒ angular position, velocity, and acceleration
- ⇒ rotational kinematics
 - right-hand rule (RHR)
- ⇒ center of mass
- ⇒ rotational energy
 - moment of inertia
 - parallel-axis theorem
- ⇒ torque
- ⇒ rotational dynamics
 - rotation about a fixed axis
 - static equilibrium
- ⇒ kinetic energy of rolling
- ⇒ cross product
 - right-hand rule (RHR)
- ⇒ the vector description of rotational motion
- ⇒ angular momentum
 - conservation of angular momentum

Chapter 12: Problem Solving

Dynamics problems for a rotating body are based on the vector equation $\vec{\tau}_{net} = d\vec{L} / dt$. Some problems simply ask for a calculation of a torque or angular momentum using the general definitions. Identify the point to be used as the origin, then evaluate the appropriate vector product. To find a torque, take \vec{r} to be the vector from the origin to the point of application of the force and evaluate $\vec{r} \times \vec{F}$. To find the angular momentum of a particle, take \vec{r} to be the vector from the origin to the particle, then evaluate $\vec{r} \times \vec{p}$ or $\vec{r} \times m\vec{v}$. You must be given the linear momentum of the particle or else its mass and velocity. To find the angular momentum of a rigid body rotating about a fixed axis, use $L = I\omega$, where I is the rotational inertia of the body and ω is its angular speed. The component of \vec{L} along the axis of rotation is the most important component and its direction is given by a right-hand rule.

Some problems can be solved using the principle of angular momentum conservation. To see if the principle can be used, you must select a system and examine the net external torque acting on it to see if it vanishes. If it does, then angular momentum is conserved. If one component of the total torque vanishes, then that component of the angular momentum is conserved, regardless of whether the other components change.

Some problems deal with an object whose rotational inertia changes at some time while it is spinning about a fixed axis. The problem statement gives information that can be used to calculate the initial values I_0 and ω_0 , before the rotational inertia changes, and asks for either I or ω at a later time, after they change. If no external torques act, angular momentum is conserved and $I_0\omega_0 = I\omega$. This can be solved for one of the quantities in terms of the others.

Mathematical Skills

You should know how to find the magnitude and direction of vector products, such as $\vec{a} \times \vec{b}$.

Recall that the magnitude is $ab\sin\phi$, where ϕ is the smallest angle between \vec{a} and \vec{b} when they are drawn with their tails at the same point. You should also remember the right hand rule for finding the direction. The product is perpendicular to the plane of \vec{a} and \vec{b} . Curl the fingers of your right hand so they rotate \vec{a} toward \vec{b} through the angle ϕ . Then, the thumb will point in the direction of the vector product.

You also need to know how to compute the vector product in terms of the components of the factors:

$$(\vec{a} \times \vec{b})_x = a_y b_z - a_z b_y \qquad (\vec{a} \times \vec{b})_y = a_z b_x - a_x b_z \qquad (\vec{a} \times \vec{b})_z = a_x b_y - a_y b_x$$

GENERAL PRINCIPLES

Solving Rotational Dynamics Problems

MODEL Model the object as a **rigid body**.

VISUALIZE Draw a pictorial representation.

SOLVE Use **Newton's second law** for rotational motion:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

Use rotational kinematics to find angles and angular velocities.

ASSESS Is the result reasonable?

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{\text{rot}} + U_G = \frac{1}{2}I\omega^2 + Mgy_{\text{cm}}$
- Rolling $E = K_{\text{rot}} + K_{\text{cm}} + U_G = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 + Mgy_{\text{cm}}$

Angular momentum is conserved if $\vec{\tau}_{\text{net}} = \vec{0}$.

- Particle $\vec{L} = \vec{r} \times \vec{p}$
- Rotation about a symmetry axis or fixed axle $\vec{L} = I\vec{\omega}$

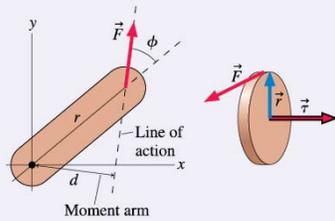
IMPORTANT CONCEPTS

Torque is the rotational equivalent of force:

$$\tau = rF \sin \phi = rF_t = dF$$

The vector description of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

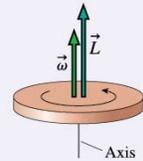


Vector description of rotation

Angular velocity $\vec{\omega}$ points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about a fixed axle or an axis of symmetry, the angular momentum is $\vec{L} = I\vec{\omega}$.

Newton's second law is $\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$.



A system of particles on which there is no net force undergoes unconstrained rotation about the **center of mass**:

$$x_{\text{cm}} = \frac{1}{M} \int x \, dm \quad y_{\text{cm}} = \frac{1}{M} \int y \, dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass M concentrated at the center of mass.

The moment of inertia

$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, I about a parallel axis distance d away is given by the **parallel-axis theorem**: $I = I_{\text{cm}} + Md^2$.

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APPLICATIONS

Rigid-body model

- Size and shape do not change as the object moves.
- The object is modeled as particle-like atoms connected by massless, rigid rods.



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Rigid-body equilibrium

An object is in total equilibrium only if both $\vec{F}_{\text{net}} = \vec{0}$ and $\vec{\tau}_{\text{net}} = \vec{0}$.

No rotational or translational motion

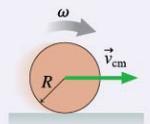


Rolling motion

For an object that rolls without slipping

$$v_{\text{cm}} = R\omega$$

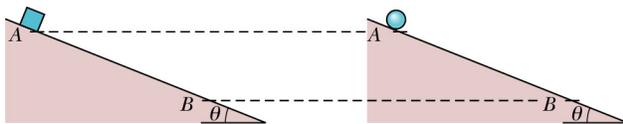
$$K = K_{\text{rot}} + K_{\text{cm}}$$



Question and Example Problems from Chapter 12

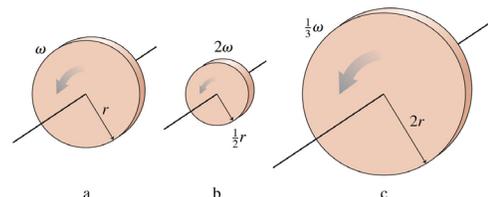
Conceptual Question 12-A

In the figure below, a block slides down a frictionless ramp and a sphere rolls without sliding down a ramp of the same angle θ . The block and sphere have the same mass, start from rest at point A, and descend through point B. (a) In that descent, is the work done by the gravitational force on the block greater than, less than, or the same as the work done by the gravitational force on the sphere? At B, which object has more (b) translational kinetic energy and (c) speed down the ramp?



Conceptual Question 12-3

The figure shows three rotating disks, all of equal mass. Rank, in order, from largest to smallest, their rotational kinetic energies K_a to K_c .



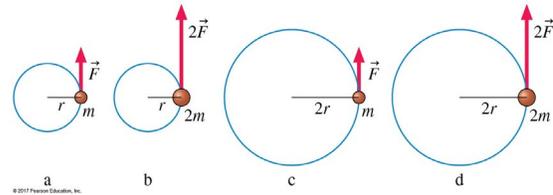
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Conceptual Question 12-7

The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims one is a solid sphere and the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?

Conceptual Question 12-10

Rank in order, from largest to smallest, the angular accelerations α_a to α_d in the figure. *Explain.*



Problem 4-41

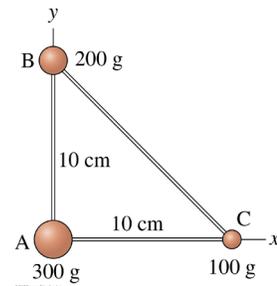
An electric fan goes from rest to 1800 rpm in 4.0 s. What is the angular acceleration?

Problem 4-43

Starting from rest, a DVD steadily accelerates to 500 rpm in 1.0 s, rotates at this angular speed for 3.0 s, then steadily decelerates to a halt in 2.0 s. How many revolutions does it make?

Problem 12-6

The three masses shown in the figure are connected by massless, rigid rods. What are the coordinates of the center of mass?

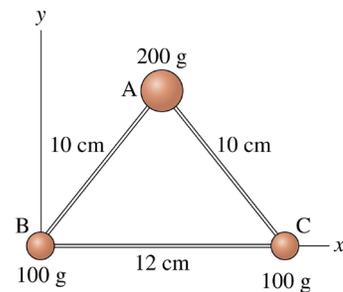


Problem 12-10

What is the rotational kinetic energy of the earth? Assume the earth is a uniform sphere. Data for the earth can be found inside the back cover of the book.

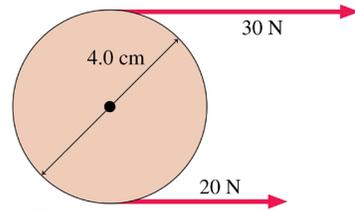
Problem 12-15

The three masses shown in the figure are connected by massless, rigid rods. (a) Find the coordinates of the center of mass. (b) Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page. (c) Find the moment of inertia about an axis that passes through masses B and C.

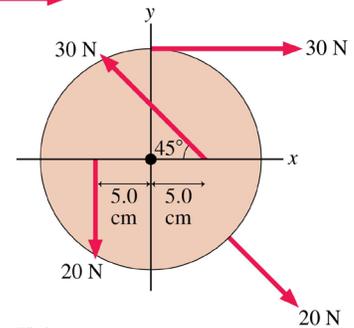


Problem 12-18

In the figure, what is the net torque about the axle?

**Problem 12-20**

The 20-cm-diameter disk in the figure can rotate on an axle through its center. What is the net torque about the axle?

**Problem 12-26**

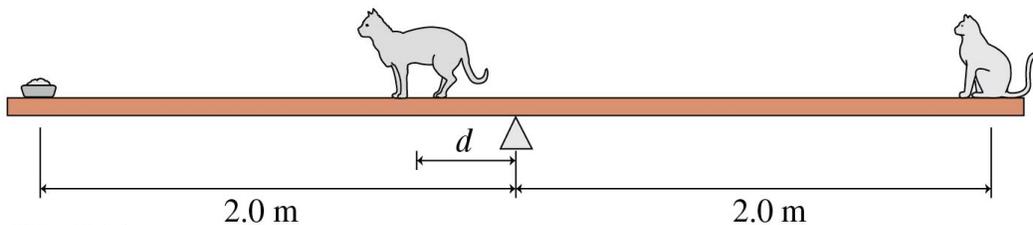
A 1.0 kg ball and a 2.0 kg ball are connected by a 1.0-m-long rigid, massless rod. The rod is rotating CW about its center of mass at 20 rpm. What net torque will bring the balls to a halt in 5.0 s?

Problem 12-28

A 4.0 kg, 36-cm-diameter metal disk, initially at rest, can rotate on an axle along its axis. A steady 5.0 N tangential force is applied to the edge of the disk. What is the disk's angular velocity, in rpm, 4.0 s later?

Problem 12-32

A 5.0 kg cat and a 2.0 kg bowl of tuna fish are at opposite ends of the 4.0-m-long seesaw of the figure below. How far to the left of the pivot point must a 4.0 kg cat stand to keep the seesaw balanced?

**Problem 12-36**

A solid sphere of radius R is placed at a height of 30 cm on a 15° slope. It is released and rolls, without slipping, to the bottom. From what height should a circular hoop of radius R be released on the same slope in order to equal the sphere's speed at the bottom.

Problem 12-39

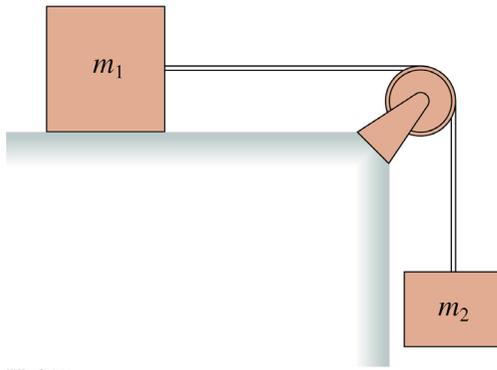
Vector $\vec{A} = 3\hat{i} + \hat{j}$ and vector $\vec{B} = 3\hat{i} - 2\hat{j} + 2\hat{k}$. What is the cross product $\vec{A} \times \vec{B}$?

Problem 12-45

How fast, in rpm, would a 5.0 kg, 22-cm-diameter bowling ball have to spin to have an angular momentum of $0.23 \text{ kg m}^2/\text{s}$?

Problem 12-65

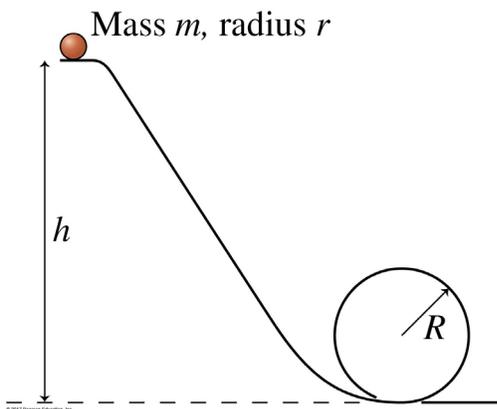
Blocks of mass m_1 and m_2 are connected by a massless string that passes over the pulley in the figure. The pulley turns on the frictionless bearings. Mass m_1 slides on a horizontal, frictionless surface. Mass m_2 is released while the blocks are at rest. **(a)** Assume the pulley is massless. Find the acceleration of mass m_1 and the tension in the string. This is a Chapter 7 review problem. **(b)** Suppose the pulley has mass m_p and radius R . Find the acceleration of m_1 and the tensions in the upper and lower portions of the string. Verify that the answers agree with part a if you set $m_p=0$.

**Problem 12-69**

A hollow sphere is rolling along a horizontal floor at 5.0 m/s when it comes to a 30° incline. How far up the incline does it roll before reversing direction?

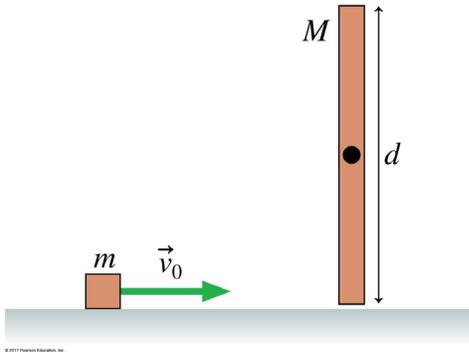
Problem 12-75

The marble rolls down the track shown in the figure and around a loop-the-loop of radius R . The marble has mass m and radius r . What minimum height h must the track have for the marble to make it around the loop-the-loop without falling off?

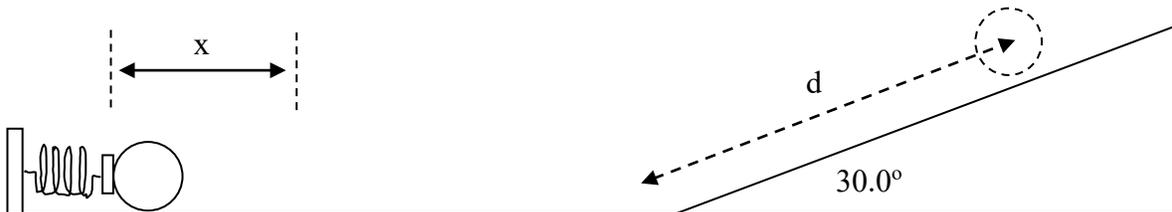


Problem 12-89

The figure shows a cube of mass m sliding without friction at speed v_0 . It undergoes a perfectly elastic collision with the bottom tip of the rod of length d and mass $M = 2m$. The rod is pivoted about a frictionless axle through its center, and initially it hangs straight down and is at rest. What is the cube's velocity – both speed and direction – after the collision?

**Problem 12-A**

A solid steel ball of mass 0.50 kg and diameter 20 cm is held in place against a spring with spring constant $k = 100 \text{ N/m}$, compressing the spring a distance $x = 30 \text{ cm}$. The ball is then released from rest and rolls without slipping along a horizontal floor. It then makes a smooth transition to an inclined plane and rolls without slipping up the plane as shown in the figure below.

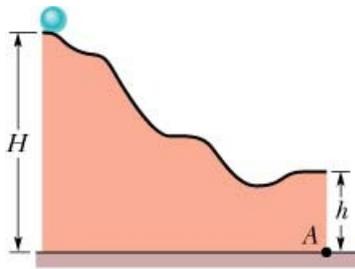


a) How far up the incline plane (i.e. what is d in the figure) does the ball roll before coming to rest?

b) How fast is the ball rolling (i.e. what is the speed of the ball's center of mass) when it has rolled 1.2 m up the inclined plane?

Problem 12-B

In the figure below, a solid ball rolls smoothly from rest (starting at height $H = 6.0$ m) until it leaves the horizontal section at the end of the track at height $h = 2.0$ m. How far horizontally from point A does the ball hit the floor?



Problem 12-C

At the instant the displacement of a 2.00 kg object relative to the origin is

$$\vec{d} = (2.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j} - (3.00\text{ m})\hat{k}$$
 its velocity is $\vec{v} = -(6.00\text{ m/s})\hat{i} + (3.00\text{ m/s})\hat{j} + (3.00\text{ m/s})\hat{k}$

and it is subject to a force $\vec{F} = (6.00\text{ N})\hat{i} - (8.00\text{ N})\hat{j} + (4.00\text{ N})\hat{k}$. Find (a) the acceleration of the object, (b) the angular momentum of the object about the origin, (c) the torque about the origin acting on the object, and (d) the angle between the velocity of the object and the force acting on the object.

Problem 12-D

In the figure below, a small 50 g block slides down the frictionless surface through height $h = 20$ cm and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point O through the angle θ before momentarily stopping. Find θ .

